

Bellwork: 11/2

$$\begin{aligned} \left(-\frac{1}{3}x + 1\right) &= \left(\frac{3}{2}x - 1\right) \\ -2x + 6 &= 9x - 6 \\ -2x &= 9x - 12 \\ -9x & \quad -9x \\ -11x &= -12 \\ -11 & \quad -11 \\ x &= \frac{12}{11} \end{aligned}$$

$$f(x) = 2x; g(x) = x^2 + 2$$

$$g(f(x)) =$$

$$g(2x)$$

$$(2x)^2 + 2$$

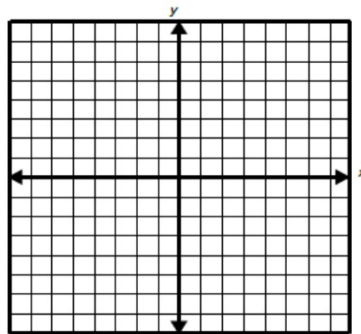
$$4x^2 + 2$$

Given the function: $f(x) = x + 3$

1.) Create an x/y chart:

x	1	2	3	4	5	6
y						

2.) Graph $f(x) = x + 3$



If we **interchange** the x and y coordinates, we form an **inverse function** of f . The notation used to denote the inverse is _____



Create an x/y chart for the $f^{-1}(x)$ and graph above. What can you conclude?

x	4	5	6	7	8	9
y						

Note: Inverse functions "undo" each other.

*The domain of $f(x)$ is the range of $f^{-1}(x)$ and vice versa.

Inverse of a Relation

- ❖ The inverse of a relation consisting of the ordered pairs (x, y) is the set of all ordered pairs (y, x)
- ❖ The domain of the inverse is the range of the original relation
- ❖ The range of the inverse is the domain of the original relation

$f(x)$ $f^{-1}(x)$

$D: \rightarrow D:$
 $R: \rightarrow R:$

Example 1: Find the inverse of each relation. State whether the relation is a function. State whether the inverse is a function.

a.) $\{(1, 2), (2, 4), (3, 6), (4, 8)\}$ **yes function**

Inverse: $\{(2, 1), (4, 2), (6, 3), (8, 4)\}$

b.) $\{(\underline{1}, 5), (\underline{1}, 6), (3, 6), (4, 9)\}$ **No** **yes function**

Inverse: $\{(5, \underline{1}), (6, \underline{1}), (\underline{6}, 3), (9, 4)\}$ **No**

$\{(1, 3), (2, 3), (3, 5)\}$ **yes**

$\{(3, 1), (3, 2), (5, 3)\}$ **no**

To find the inverse of a function:

- 1.) Replace $f(x)$ notation with y
- 2.) Rewrite function by interchanging x and y
- 3.) Solve for y
- 4.) Replace y with $f^{-1}(x)$

Example 2: $f(x) = \frac{5-3x}{2}$

$$\frac{2x-5}{-3} = f^{-1}(x)$$

To find the inverse of a function:

- 1.) Replace $f(x)$ notation with y
- 2.) Rewrite function by interchanging x and y
- 3.) Solve for y
- 4.) Replace y with $f^{-1}(x)$

Example 3: $f(x) = \frac{4-x}{6}$

$$f^{-1}(x) = -6x + 4$$

$$y = \frac{4-x}{6}$$

$$6 \cdot x = \left(\frac{4-y}{6} \right) 6$$

$$6x = 4 - y$$

$$6x - 4 = -y$$

$$\frac{6x-4}{-1} = y$$

$$\frac{6x-4}{-1}$$

$$\underline{f^{-1}(x) = -6x + 4}$$

To find the inverse of a function:

- 1.) Replace $f(x)$ notation with y
- 2.) Rewrite function by interchanging x and y
- 3.) Solve for y
- 4.) Replace y with $f^{-1}(x)$

Example 4: $f(x) = 10x - 5$

$$\frac{x+5}{10} = f^{-1}(x)$$

Verifying That Functions are Inverses

Two functions are inverses if:

$$f \circ f^{-1}(x) = x \quad \text{and} \quad f^{-1} \circ f(x) = x$$

Determine whether the following functions are inverses using composition.

4.) $f(x) = 4x + 1$ $g(x) = \frac{x-1}{4}$

Verifying That Functions are Inverses

Two functions are inverses if:

$$f \circ f^{-1}(x) = x \quad \text{and} \quad f^{-1} \circ f(x) = x$$

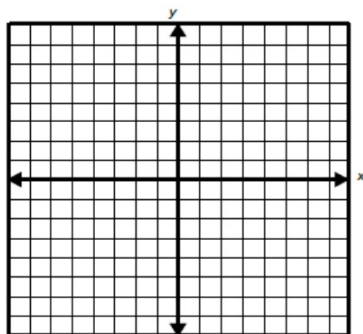
5.) $f(x) = x - 3$ $g(x) = -x + 3$

Determining if the inverse of a function is a function from a graph:

Horizontal-Line Test

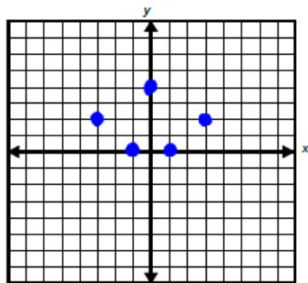
- ❖ The inverse of a function is a function if and only if every horizontal line intersects the graph of the given function at no more than one point.

Example 5: Graph $f(x) = 3x - 2$. Determine whether its inverse is a function. Then find the inverse and graph it to verify.

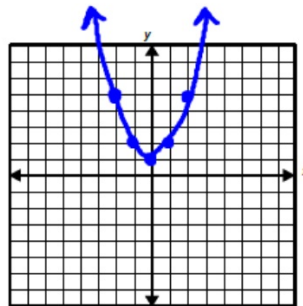


If a function has an inverse that is also a function, then the function is a **one-to-one function**. Therefore, every 1-1 function passes the horizontal-line test.

Is the relation a function? Find the domain and range. Graph the inverse of each relation and determine whether the inverse is a function. Then find the domain and range of the inverse.

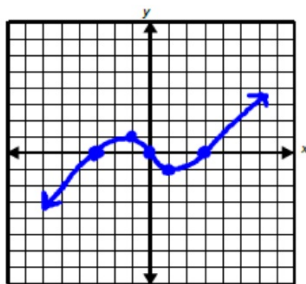


Original:	Inverse:
F? _____	F? _____
D: _____	D: _____
R: _____	R: _____

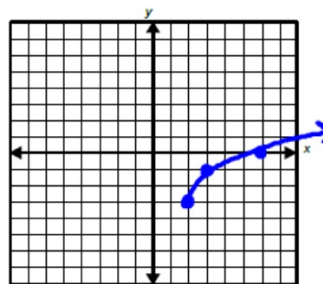


Original:	Inverse:
F? _____	F? _____
D: _____	D: _____
R: _____	R: _____

Is the relation a function? Find the domain and range. Graph the inverse of each relation and determine whether the inverse is a function. Then find the domain and range of the inverse.



Original:	Inverse:
F? _____	F? _____
D: _____	D: _____
R: _____	R: _____



Original:	Inverse:
F? _____	F? _____
D: _____	D: _____
R: _____	R: _____